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HEAT CONDUCTION BETWEEN BODIES WITH WAVY SURFACES

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Abstract—Theoretical treatment is provided for the conductive heat transfer between two solids with wavy surfaces. The problem is set in two dimensions, and it is assumed that the surface profiles are purely sinusoidal and deformations elastic. It is also assumed that heat is transmitted only where there is solid to solid contact, and there is no resistance due to contamination of the surfaces. The temperature problem is solved exactly, but deformations are treated by relying on the results due to Hertz. The expression derived for the constriction resistance exhibits a directional effect. It is also shown that perfectly flat surfaces may become wavy, as heat is transmitted from one body to another.

NOMENCLATURE

b, amplitude of sinusoidal gap;
$\bar{c}$, half length of contact;
c, normalized half length of contact;
H, Heaviside step function;
k, thermal conductivity;
l, half wave length of surface profile;
p, applied pressure;
P, force in Hertz formula;
P_n, Legendre polynomial of order n;
q, heat flux;
R, dimensionless constriction resistance;
S, uniaxial compliance in plane strain;
T, temperature;
$\bar{x}, \bar{y}$, cartesian coordinates;
x, y, normalized cartesian coordinates.

Greek symbols

α, coefficient of thermal expansion;
δ, distortivity;
κ, curvature;
Λ, dimensionless heat flux;
μ, shear modulus;
ν, Poisson's ratio;
Π, dimensionless applied pressure;
ρ, constriction resistance.

INTRODUCTION

TRANSFER of heat between two solids by conduction is a problem of some fundamental and technological importance, and it has accordingly received considerable attention over the last two or three decades. While not overwhelming, the amount of literature on this topic is extensive, and it would be out of place here to attempt summarizing the assumptions and approaches used in previous theoretical considerations and the results achieved. It might be said without implying criticism of any kind, however, that in seeking to include all physically important aspects of the problem, analytical tractability has largely been forsaken. It is generally recognized, for instance, that thermal contact resistance is intimately connected with surface roughness and the elastic or plastic deformations of the asperities [1]. A characterization of the rough surfaces can be done only on a statistical basis, but once a statistical description is injected, it becomes virtually impossible to solve the pertinent elasticity and plasticity problems.

Exactly the opposite approach is used in the present article to discuss the conductive heat transfer between two contacting bodies with nominally flat surfaces, and the problem is remorselessly idealized to the point where it is feasible to solve the full heat conduction and elasticity equations. First, the problem is set in two dimensions. Second, the gap between the two bodies in their undeformed state is taken as purely sinusoidal with a wave length that is large in comparison to the amplitude. Furthermore, the boundary conditions are satisfied not on the actual surfaces, but rather on the nominal plane of contact. If this is done, it makes no difference what the actual surface profiles are, and only the gap enters the formulation. Thus Fig. 1 shows one of the bodies having a perfectly flat surface, while the surface of the other body is sinusoidal. It is also assumed that heat is transmitted only where there is solid to solid contact, that radiation and the effect of an intervening fluid are insignificant, and that there is no resistance due to an oxide film or other contamination of the surfaces. Finally it is supposed that the global

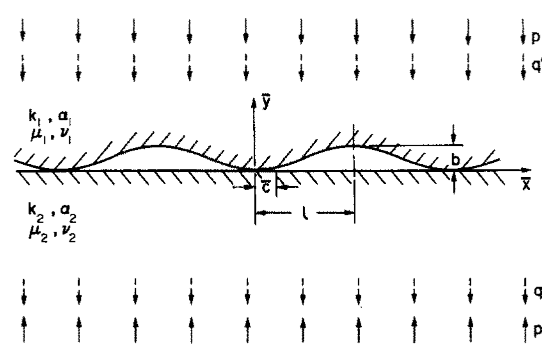

FIG. 1. Geometry of the problem.

warping of the two bodies is suppressed by applying suitable forces far away.

The two solids are pressed together, as indicated in Fig. 1, so that the initial point contacts spread over finite intervals, and steady state heat conduction is established by maintaining a suitable temperature difference. The principal problem is to relate the constriction resistance to the applied pressure, rate of heat flow and the material constants of the two bodies. It is convenient in the analysis to view the extent of contact $\bar{c}$ as a fixed parameter and to compute the combination of the applied pressure p and heat flux q^{∞} that lead to the given $\bar{c}$. The excess temperature difference required to drive a given q^{∞} through the interface with periodic contacts in comparison to full contact can also be calculated for a specified $\bar{c}$, and thus the constriction resistance related to p and q^{∞}. The associated boundary value problems involve mixed conditions. However, taking advantage of the periodic nature of the fields, they can be reduced to dual series equations which yield to standard techniques.

TEMPERATURE DISTRIBUTION

As indicated in Fig. 1, the half wave length of the sinusoidal gap is denoted by l, the amplitude by b and the extent of contact by $2\bar{c}$. It is expedient to introduce the dimensionless coordinates and extent of contact

$$x = \pi\bar{x}/l, \quad y = \pi\bar{y}/l, \quad c = \pi\bar{c}/l \qquad (1)$$

and to use subscripts 1 and 2 for reference to the two bodies occupying the regions $y > 0$ and $y < 0$, respectively. Under steady state conditions, the temperature distribution must be harmonic in the coordinates x, y. The boundary conditions along the nominally flat contact interface $y = 0$ are

$$k_1\frac{\partial T_1}{\partial y} = k_2\frac{\partial T_2}{\partial y}, \qquad 0 \leqslant x \leqslant \pi, \qquad (2)$$

$$T_1 = T_2 = 0, \qquad 0 \leqslant x < c, \qquad (3)$$

$$\frac{\partial T_1}{\partial y} \text{ or } \frac{\partial T_2}{\partial y} = 0, \quad c < x \leqslant \pi, \qquad (4)$$

where k denotes thermal conductivity. The second boundary condition (3) may require an explanation: The temperature at the origin $x = y = 0$ can be set equal to zero by adjusting the general temperature level. It can be reasoned then [2] that the temperature must vanish along the whole contact interval $0 \leqslant x < c$, $y = 0$.

The harmonic functions which are suitable for solving the heat conduction problem are

$$T_1 = \frac{q^{\infty}l}{\pi k_1}\left(y + A_0 + \sum_{n=1}^{\infty} A_n \mathrm{e}^{-ny}\cos nx\right) \qquad (5)$$

$$T_2 = \frac{q^{\infty}l}{\pi k_2}\left(y + B_0 + \sum_{n=1}^{\infty} B_n \mathrm{e}^{ny}\cos nx\right). \qquad (6)$$

It may be noted that these temperature distributions have already been adjusted so that they yield consistent heat flow rates at infinity, and that the series have the correct periodicity to reflect the alternating pattern of contacts and gaps.

Boundary condition (2) gives upon substitution

$$B_n = -A_n, \quad n = 1, 2, \ldots. \qquad (7)$$

The first of the split conditions (3) leads to

$$B_0 = -A_0 \qquad (8)$$

and

$$A_0 + \sum_{n=1}^{\infty} A_n \cos nx = 0, \quad 0 \leqslant x < c. \qquad (9)$$

Finally the second of the split conditions (4) results in

$$\sum_{n=1}^{\infty} nA_n \cos nx = 1, \quad c < x \leqslant \pi. \qquad (10)$$

The solution of the dual series equations (9) and (10) for the unknown coefficients A_n $(n = 0, 1, 2, \ldots)$ is explained in the Appendix. The resulting temperature distribution from (5) and (6) is

$$T_1 = \frac{q^{\infty}l}{\pi k_1}\left\{y - 2\log(\sin\tfrac{1}{2}c) - \sum_{n=1}^{\infty}\frac{1}{n}[P_n(\cos c) + P_{n-1}(\cos c)]\mathrm{e}^{-ny}\cos nx\right\} \qquad (11)$$

$$T_2 = \frac{q^{\infty}l}{\pi k_2}\left\{y + 2\log(\sin\tfrac{1}{2}c) + \sum_{n=1}^{\infty}\frac{1}{n}[P_n(\cos c) + P_{n-1}(\cos c)]\mathrm{e}^{ny}\cos nx\right\} \qquad (12)$$

where P_n denotes the Legendre polynomial of order n. It is also shown in the Appendix that the temperature distribution along the interface $y = 0$ can be reduced to a closed form:

$$(T_1)_{y=0} = \frac{2q^{\infty}l}{\pi k_1}H(x-c)\log[R + \sqrt{(R^2-1)}] \qquad (13)$$

$$(T_2)_{y=0} = -\frac{2q^{\infty}l}{\pi k_2}H(x-c)\log[R + \sqrt{(R^2-1)}] \qquad (14)$$

where $0 \leqslant x \leqslant \pi$, H denotes the Heaviside step function and

$$R = \frac{\sin\frac{1}{2}x}{\sin\frac{1}{2}c} \geqslant 1. \qquad (15)$$

The temperature profiles along a set of x = constant lines are shown in Fig. 2 for several extents of contact c.

DEFORMATIONS

In order to relate the extent of contact $\bar{c}$ to the applied pressure p and the far field heat flux q^{∞} it is necessary to solve a mixed boundary value problem in thermoelasticity. Although this problem can be reduced to a Fredholm integral equation which is amenable to a numerical solution, the procedure and results are quite complicated. An asymptotic analysis of the thermoelastic solution reveals, however, that accurate results for $\bar{c} < 0.3l$ can be obtained by simply applying the well known theory of Hertz for contact between two bodies with cylindrical surfaces. The condition of $\bar{c} < 0.3l$ is not very restrictive if the deformations are

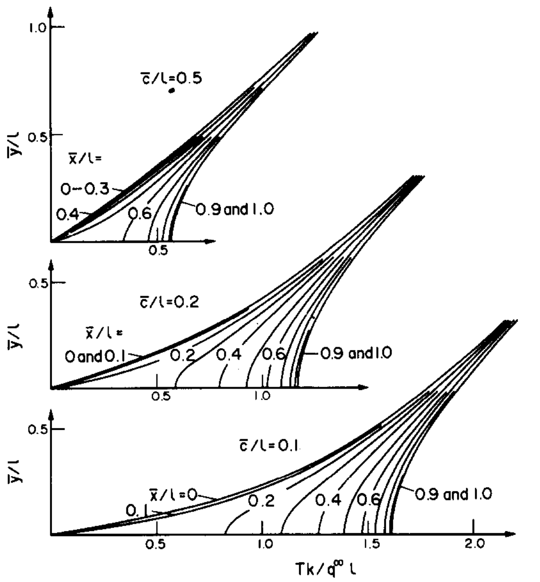

FIG. 2. Temperature profiles for different extents of contact.

assumed to be purely elastic. If for instance $q^\infty = 0$ and the two materials have the same elastic constants, the applied pressure required to achieve $\bar{c} = 0.3$ is approximately $\frac{1}{4}\mu(b/l)$ and the maximum contact pressure is roughly $\mu(b/l)$ [3]. Unless the ratio b/l is extremely small, most materials can therefore be expected to deform plastically before the extent of contact $\bar{c} = 0.3l$ is reached, if the isothermal solution is accepted as a yardstick for making order of magnitude assessments.

The Hertz formula for two cylinders that are pressed together is [4]

$$\bar{c}^2 = \frac{4P(S_1+S_2)}{\pi(\kappa_1+\kappa_2)} \tag{16}$$

where $\bar{c}$ is the half length of contact and P the transmitted force per unit length of the cylinders. Furthermore, κ denotes the local curvature counted positive if the cylinder is convex to the outside, and S is the uniaxial compliance in plane strain. In terms of the shear modulus μ and Poisson's ratio ν,

$$S = \frac{1-\nu}{2\mu}. \tag{17}$$

In the present problem,

$$P = 2pl. \tag{18}$$

The quantity $(\kappa_1+\kappa_2)$ in (16) constitutes a mismatch in curvatures as the two cylinders are viewed from one side. In the absence of heat flow,

$$\kappa_1+\kappa_2 = \frac{\pi^2 b}{2l^2} \tag{19}$$

as is readily computed from the equation for the sinusoidal surface profile. The curvature of a boundary is modified, however, by a change in temperature and heat flow through the boundary. For plane strain and steady state temperature distribution ($\nabla^2 T = 0$), the change in curvature is [5, 6]

$$\Delta\kappa = -\alpha(1+\nu)\left(\frac{1}{k}q_n+\kappa T\right) \tag{20}$$

where α is the coefficient of thermal expansion, q_n the heat flux through the boundary counted positive for flow out of the material and T the temperature change. Both the original curvature of the boundary κ and its change $\Delta\kappa$ are reckoned positive for convexity to the outside. It may be noted that $\Delta\kappa$ is determined by the local values of heat flux and temperature change, and that it does not matter what the overall temperature distribution is.

The heat flux through the contact regions is highly nonuniform, and there is some question regarding which particular value should be used for q_n in (20). If there were no interference from the flow through the adjacent contacts, the distribution of heat flux in the contact region would be

$$q_n(\bar{x}) = \frac{2q^\infty l}{\pi\sqrt{(\bar{c}^2-\bar{x}^2)}}. \tag{21}$$

The value at the center of the contact region

$$q_n(0) = \frac{2}{\pi}\frac{q^\infty l}{\bar{c}} \doteq 0.64\frac{q^\infty l}{\bar{c}} \tag{22}$$

is clearly an underestimate. The average value

$$\bar{q}_n = \frac{q^\infty l}{\bar{c}} = 1.00\frac{q^\infty l}{\bar{c}} \tag{23}$$

is likely to overestimate the effect, because the curvature change near the middle of the contact region is bound to be more important than that near the ends. In the absence of a better idea, a compromise between (22) and (23) is $0.82\, q^\infty l/\bar{c}$. The asymptotic form of the full thermoelasticity solution shows that the correct value for substitution in (20) is $q_n = 8q^\infty l/\pi^2\bar{c} \doteq 0.81\, q^\infty l/\bar{c}$. Using this value, the additional mismatch in curvatures caused by heat flow is

$$\kappa_1+\kappa_2 = \frac{8lq^\infty(\delta_2-\delta_1)}{\pi^2\bar{c}} \tag{24}$$

where

$$\delta = \frac{\alpha(1+\nu)}{k}. \tag{25}$$

The constant δ defined by (25) may be called the distortivity of the material, because it relates the distortion of a straight boundary to the local value of heat flux.

Adding the contributions (19) and (24) and substituting them together with (18) into the Hertz formula leads to the following quadratic equation for the determination of the extent of contact:

$$\frac{\pi^3 b}{2l^2}\bar{c}^2 + \frac{8l}{\pi}q^\infty(\delta_2-\delta_1)\bar{c} - 8pl(S_1+S_2) = 0 \tag{26}$$

or

$$c^2 - 2Aq^\infty(\delta_1-\delta_2)c - B = 0 \tag{27}$$

where

$$A = \frac{8l^2}{\pi^3 b} \geqslant 0, \quad B = \frac{16pl(S_1+S_2)}{\pi b} \geqslant 0. \tag{28}$$

The solution of (27) is

$$c = Aq^{\infty}(\delta_1-\delta_2)+\sqrt{\{[Aq^{\infty}(\delta_1-\delta_2)]^2+B\}}. \tag{29}$$

Whereas the constants A and B are non-negative, the term $q^{\infty}(\delta_1-\delta_2)$ changes sign upon reversal of heat flow or interchange of materials. As seen from (29), the extent of contact c depends consequently on the direction of heat flow unless $\delta_1-\delta_2 = 0$.

CONSTRICTION RESISTANCE AND DIRECTIONAL EFFECT

The first terms in (11) and (12), which are linear in y, correspond to uniform heat flow in two bodies that are in perfect contact. The series terms in (11) and (12) constitute local disturbances that are due to the periodic pattern of contacts and gaps. Their effect is negligible at distances from the interface that are large in comparison to the wave length of the surface profile. The second terms in (11) and (12), which are constant, give the additional temperature difference ΔT that must be supplied to drive through the wavy interface the same amount of heat as flows in two bodies with perfect contact.

The constriction resistance of the interface is defined as

$$\rho = \Delta T/q^{\infty} \tag{30}$$

and substituting for ΔT the sum of the second terms in (11) and (12)

$$\rho = -\frac{2l}{\pi}\left(\frac{1}{k_1}+\frac{1}{k_2}\right)\log(\sin\tfrac{1}{2}c). \tag{31}$$

The resistance ρ can be related to the applied pressure p and heat flux q^{∞} by first computing c from (29) for given values of p and q^{∞}, and then substituting c into (31). The results are displayed in Fig. 2 as a dimensionless plot, in which

$$R = \frac{\rho}{l}\left(\frac{1}{k_1}+\frac{1}{k_2}\right)^{-1}, \quad \Lambda = \frac{q^{\infty}l^2}{b}(\delta_1-\delta_2),$$

$$\Pi = \frac{pl}{b}(S_1+S_2). \tag{32}$$

The general trend seen from the curves is that the constriction resistance decreases with increasing applied pressure.

Another noteworthy feature of the results is that, for materials with different distortivities, the wavy interface exhibits a pronounced directional effect. As mentioned before, the extent of contact depends on the direction of the imposed heat flux. The constriction resistance, being directly related to the extent of contact, then also changes upon reversal of the direction in which heat is flowing. Figures 3 shows that the directional effect is stronger at low applied pressures and high heat fluxes. In general, the constriction resistance is lower for heat flowing into the material with the lower distortivity.

A very simple formula for the constriction resistance

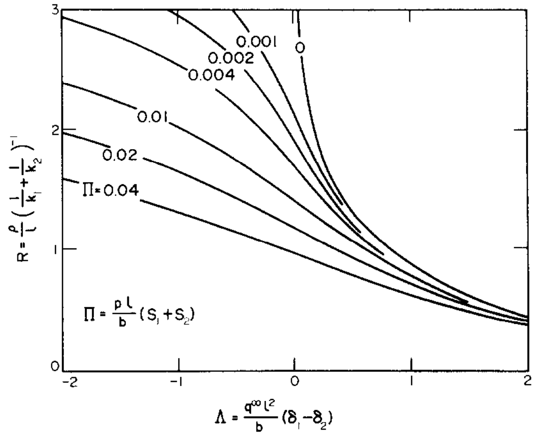

FIG. 3. Interface resistance vs heat flux for different applied pressures.

follows from (29) and (31) if the two bodies are made of the same material. Approximating $\sin\frac{1}{2}c$ by $\frac{1}{2}c$,

$$\rho = \frac{2l}{\pi k}\log\frac{\pi b}{8plS}. \tag{33}$$

CONTACT BETWEEN PERFECTLY FLAT SURFACES AND LACK OF UNIQUENESS

Suppose that the amplitude b of the wavy surface profile is allowed to approach zero in (26). In such case, (26) yields the extent of contact

$$\bar{c} = \frac{\pi p(S_1+S_2)}{q^{\infty}(\delta_2-\delta_1)}. \tag{34}$$

This result indicates that bodies with perfectly flat surfaces may not only have the linear temperature distribution that corresponds to full contact, but can also go into the steady state described by (11), (12) and (34). Since $\bar{c}$ must be positive, this is possible only for $q^{\infty}(\delta_2-\delta_1) > 0$ or the case when heat flows into the material with the higher distortivity. The second state may physically be induced by some random fluctuations in thermal conditions during the process of approaching the steady state or deviations from perfectly uniform material properties. Such non-uniformities may always be expected to be present. The result also indicates that the contact between perfectly flat surfaces has no unique solution, when heat flows into the material with the higher distortivity.*

It may be noted that the wavelength l associated with the second possible state remains arbitrary. Assuming that all Fourier components are present in the initial fluctuations that lead to the periodic steady state, the wave length to evolve is likely to be that of the fastest growing disturbance in the surface displacement. However, an analysis of the unsteady state is outside the scope of this article.

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*The possible lack of uniqueness in thermoelastic contact problems has been discussed by Barber [2].

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APPENDIX

The dual series equations may be cast into a standard form by the substitutions

$$x = \pi - \xi, \quad c = \pi - \gamma \tag{A1}$$

$$A_0 = \tfrac{1}{2}a_0, \quad A_n = (-1)^n a_n \tag{A2}$$

which change (9) and (10) to

$$\sum_{n=1}^{\infty} n a_n \cos n\xi = 1, \quad 0 \leqslant \xi < \gamma \tag{A3}$$

$$\tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos n\xi = 0, \quad \gamma < \xi \leqslant \pi. \tag{A4}$$

Following Sneddon [7],* the unknown coefficients a_n $(n = 0, 1, 2, \ldots)$ can be found from an auxiliary function which for the dual series equations (A3) and (A4) is

$$h_1(t) = \frac{2}{\pi}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t \frac{s \sin \frac{1}{2}s \, \mathrm{d}s}{\sqrt{(\cos s - \cos t)}}. \tag{A5}$$

With the change of variable $s = 2r$, the integral in (A5) reduces to a known form,† and

$$h_1(t) = \frac{\sqrt{(2)}\sin t}{1+\cos t} = \sqrt{(2)}\tan \tfrac{1}{2}t. \tag{A6}$$

*See section 5.4.3.

†See No. 3, 3.842–3.847 in [8].

The coefficients in the dual series equations are computed from the auxiliary function as

$$a_0 = \frac{2}{\sqrt{2}}\int_0^{\gamma} h_1(t)\,\mathrm{d}t = -4\log(\cos\tfrac{1}{2}\gamma) \tag{A7}$$

$$a_n = \frac{1}{\sqrt{2}}\int_0^{\gamma} h_1(t)\,[P_n(\cos t) + P_{n-1}(\cos t)]\,\mathrm{d}t$$

$$= \frac{1}{n}[P_{n-1}(\cos\gamma) - P_n(\cos\gamma)]. \tag{A8}$$

Returning to the original variables, it follows from (A1) and (A2) that

$$A_0 = -2\log(\sin\tfrac{1}{2}c) \tag{A9}$$

$$A_n = -\frac{1}{n}[P_n(\cos c) + P_{n-1}(\cos c)] \tag{A10}$$

which lead to the temperature distribution (11) and (12).

The temperature along the interface $y = 0$ in the upper half plane is

$$(T_1)_{y=0} = \frac{q^{\infty} l}{\pi k_1}\left\{A_0 - \sum_{n=1}^{\infty}\frac{1}{n}[P_n(\cos c) + P_{n-1}(\cos c)]\cos nx\right\} \tag{A11}$$

and

$$\left(\frac{\partial T_1}{\partial x}\right)_{y=0} = \frac{q^{\infty} l}{\pi k_1}\sum_{n=1}^{\infty}[P_n(\cos c) + P_{n-1}(\cos c)]\sin nx. \tag{A12}$$

The series in (A12) can be summed,* and

$$\left(\frac{\partial T_1}{\partial x}\right)_{y=0} = \frac{\sqrt{(2)}\,q^{\infty} l}{\pi k_1}\frac{\cos\frac{1}{2}x\,H(x-c)}{\sqrt{(\cos c - \cos x)}}. \tag{A13}$$

Integrating,

$$(T_1)_{y=0} = \frac{2q^{\infty} l}{\pi k_1}H(x-c)\int_c^x \frac{\mathrm{d}\sin\frac{1}{2}t}{\sqrt{(\sin^2\frac{1}{2}t - \sin^2\frac{1}{2}c)}} \tag{A14}$$

because $T_1(0,0) = 0$. The integral in (A14) is elementary, and (13) and (14) follow directly.

*See [7], p. 59.

CONDUCTION THERMIQUE ENTRE SOLIDES PRESENTANT UNE SURFACE ONDULEE

Résumé—On présente un traitement théorique du transfert de chaleur par conduction entre deux solides présentant une surface ondulée. Le problème est formulé dans deux dimensions et on suppose que le profil des surfaces est purement sinusoïdal et les déformations élastiques. On suppose également que la chaleur est transmise seulement aux points de contact des deux solides, et qu'il n'y a pas de résistance dûe aux impuretés de surface. Le problème thermique est résolu exactement, mais les déformations sont traitées en s'appuyant sur les résultats de Hertz. L'expression obtenue pour la résistance à la striction présente un effet directionnel. On montre également que des surfaces parfaitement planes peuvent devenir ondulées lorsque un échange de chaleur s'établit entre les deux solides.

WÄRMELEITUNG ZWISCHEN KÖRPERN MIT WELLIGEN OBERFLÄCHEN

Zusammenfassung—Der Vorgang der Wärmeleitung zwischen zwei Festkörpern mit welligen Oberflächen wird theoretisch behandelt. Das Problem wird zweidimensional angenommen und es ist vorausgesetzt, daß die Oberflächenprofile rein sinusförmig und deformationselastisch sind. Es wird weiter vorausgesetzt, daß Wärme nur an Festkörper/Festkörper-Kontaktstellen übertragen wird und daß kein Widerstand infolge von Verunreinigung der Oberflächen auftritt. Die Temperaturverteilung ist exakt gelöst, die Deformationen werden aufgrund von Ergebnissen von Hertz behandelt. Der für den Einschnürungswiderstand abgeleitete Ausdruck zeigt einen Richtungseffekt. Es wird auch gezeigt, daß eine vollkommen ebene Oberfläche dann wellig werden kann, wenn Wärme von einem Körper auf einen anderen übertragen wird.

КОНДУКТИВНЫЙ ТЕПЛООБМЕН МЕЖДУ ТЕЛАМИ С ВОЛНИСТОЙ ПОВЕРХНОСТЬЮ

Аннотация — Теоретически рассматривается кондуктивный теплообмен между двумя твердыми телами с волнистой поверхностью. Задача представлена как двумерная при допущении, что профили поверхности чисто синусоидальные, а деформации — упругие. Кроме того, сделано предположение, что теплообмен происходит только там, где имеется контакт твёрдых тел и отсутствует сопротивление за счёт загрязнения поверхностей. Тепловая задача решена точно, а деформации рассматриваются исходя из результатов Герца. Выражение, выведенное для сопротивления при сжатии, обнаруживает эффект направленности. Кроме того, показано, что абсолютно плоские поверхности могут стать волнистыми по мере того, как тепло передается от одного тела к другому.